Parametric study of torsional response of phased accelerograms - 8CCEE

Alexander, N.A.[1], Javed, K.[2], Noor, F.N.[3], Goorvadoo, N[4]

ABSTRACT

For buildings with two eccentricities there exists, in the literature, several different definitions of the uncoupled torsional to lateral frequency ratio. This study clarifies the discrepancies and define the inter-relationship between these various definitions. Under parametric analysis an arbitrary selection of eccentricities and system frequencies can result in an physically inadmissible structure. Theoretical investigation, by use of Gershgorin's theorem, is required to establish bounds for these system parameters. Relational formulae are developed for different definitions of the uncoupled torsional to lateral frequency ratio. Graphical descriptions of admissibility bounds on system parameters are produced. In many past studies the phase spectrum of the ground acceleration and more interestingly the phase difference spectrum between orthogonal horizontal ground motions has been neglected. Investigation of the effect of ground motion phase difference on the structural response of a structural system is performed. Non-linear time history analysis is used in conjunction with Fourier spectral methods. The cross-correlation function is used. The variation in the response quantities with respect to the phase content of accelerogram has been found to be significant. Evidence of phase-difference spectrum to torsional mode amplification relationship is observed.

INTRODUCTION

The dynamics of structures has, traditionally, been investigated solely for the amplitude of the ground acceleration. The accelerograms also contain phase information of the component frequencies. Under bi-directional loading, the phase difference in the two lateral components of accelerograms should affect the structural response. Particular peaks of the ground motion in one horizontal direction may or may not be synchronised with peaks in the other orthogonal direction. Models of single storey asymmetric structures with lumped mass idealisation, are studied under the phased ground excitation. Three degrees of freedom, (x_o, y_o, θ) two lateral and one twisting in the horizontal plane, are considered. For a single storey idealisation the (CR) is the centre of rigidity. (Hejal, R. *et al* 1987) defines clearly the location and nature of the CR. The stiffness actions applied at the CR while the inertial actions apply at the Centre of Mass (CM). By considering force and torque equilibrium about an arbitrary Origin O on the floor, then applying a co-ordinate transformation, the resulting equation of motion is given by

$$\begin{bmatrix} m & 0 & -my_G \\ 0 & m & mx_G \\ -my_G & mx_G & m(r_m^2 + x_G^2 + y_G^2) \end{bmatrix} \begin{bmatrix} \ddot{x}_o \\ \ddot{y}_o \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} \Sigma k_{xi} & 0 & -\Sigma y_i k_{xi} \\ 0 & \Sigma k_{yi} & \Sigma x_i k_{yi} \\ -\Sigma y_i k_{xi} & \Sigma x_i k_{yi} & \Sigma(x_i^2 k_{yi} + x_i^2 k_{yi} + k_{\theta i}) \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ \theta \end{bmatrix} = -\begin{bmatrix} m\ddot{x}_g \\ m\ddot{y}_g \\ -m\ddot{x}_g y_G - m\ddot{y}_g x_G \end{bmatrix} \quad (1)$$

where the m is the lumped floor mass, $(k_{xi}, k_{yi}, k_{\theta i})$ are the lateral bending stiffnesses and the torsional stiffness of the *ith* element (columns etc.). (x_i, y_i) are the co-ordinates of the *ith* element from O. (x_G, y_G) are the co-ordinates of the CM from O. r_m is the radius of gyration about the CM. Equation (1) is parameterised and normalised by the introduction of eccentricity ratios and frequency parameters or frequency ratio parameters. The origin O is conventionally taken to be either CR or CM and this results in different forms of equation (1) with different definitions for the system parameters. There are at least two possible mass polar moments of inertia definitions (a) $J_m = mr_m^2$ (b) $J_r = mr_r^2$ where r_r is the radius of gyration about the CR. Similarly there at least two possible floor torsional stiffness definitions

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[1] Senior Lecturer, Department of Civil Engineering, University of East London. UK.
[2] Research Student, Department of Civil Engineering, University of East London. UK.
[3] Principal Lecturer, Department of Civil Engineering, University of East London. UK.
[4] Research Student, Department of Civil Engineering, University of East London, UK.

(a) $K_{mt} = \sum (x_{mi}^2 k_{yi} + y_{mi}^2 k_{xi} + k_{\theta i})$ where (x_{mi}, y_{mi}) are element co-ordinates from CM, or (b) $K_{rt} = \sum (x_{ri}^2 k_{yi} + y_{ri}^2 k_{xi} + k_{\theta i})$ where (x_{ri}, y_{ri}) are element co-ordinates from CR. Note that K_{rt} is the actual floor static torsional stiffness at the CR ie the torque required to produce a unit rotation about the CR. However K_{mt} is not strictly speaking the floor 'torsional stiffness defined at the CM' since under the action of a static torque the only point on the floor that rotates but does not displace is the CR. The introduction of angular displacement variables simplifies the equation of motion but can be achieved by using either r_r or r_m. Note that use of r_m would seem favourable in most cases as it is independent of eccentricities which will vary under inelastic conditions. Hence when introducing the 'uncoupled' torsional frequency ratio there are at least four possible definitions

$$\lambda_{mm}^2 = \frac{K_{mt}}{r_m^2 K_x}, \quad \lambda_{mr}^2 = \frac{K_{mt}}{r_r^2 K_x}, \quad \lambda_{rm}^2 = \frac{K_{rt}}{r_m^2 K_x}, \quad \lambda_{rr}^2 = \frac{K_{rt}}{r_r^2 K_x} \tag{2}$$

where $\left(K_x = \sum k_{xi}, K_y = \sum k_{yi}\right)$. This leads to some confusion when results are compared by different authors. (Chandler, A.M. *et al* 1986), (Tso, W.K. *et al* 1986) and others use λ_{rr} while (Goel, R.K. *et al* 1991) use λ_{rm} and (Hejal, R. *et al* 1987) uses λ_{mm}. Eccentricities $\left(\varepsilon_x = \sum x_{mi} k_{yi} / \sum k_{yi}, \varepsilon_y = \sum y_{mi} k_{xi} / \sum k_{xi}\right)$ are the co-ordinates of the CR from the CM. Hence eccentricity ratios can be either $(\varepsilon_{mx}, \varepsilon_{my}) = (\varepsilon_x / r_m, \varepsilon_y / r_m)$ or $(\varepsilon_{rx}, \varepsilon_{ry}) = (\varepsilon_x / r_r, \varepsilon_y / r_r)$. By applying perpendicular and parallel axes theorems the following relations can be derived.

$$r_r^2 = r_m^2 + \varepsilon_x^2 + \varepsilon_y^2, \quad \varepsilon_{rx}^2 = \frac{\varepsilon_{mx}^2}{1 + \varepsilon_{mx}^2 + \varepsilon_{my}^2}, \quad \varepsilon_{ry}^2 = \frac{\varepsilon_{my}^2}{1 + \varepsilon_{mx}^2 + \varepsilon_{my}^2} \tag{3}$$

Generally speaking the eccentricity ratios defined at the CR are larger than those defined at CM. By co-ordinate transforms the following results can be derived linking the various definitions of the 'uncoupled' torsional frequency parameter.

$$K_{rt} = K_{mt} - \varepsilon_x^2 K_y - \varepsilon_y^2 K_x, \quad \lambda_{rm}^2 = \lambda_{mm}^2 - \varepsilon_{my}^2 - \lambda_y^2 \varepsilon_{mx}^2, \quad \lambda_{rr}^2 = \frac{\lambda_{mm}^2 - \varepsilon_{my}^2 - \lambda_y^2 \varepsilon_{mx}^2}{1 + \varepsilon_{mx}^2 + \varepsilon_{my}^2}, \quad \lambda_{mr}^2 = \frac{\lambda_{mm}^2}{1 + \varepsilon_{mx}^2 + \varepsilon_{my}^2} \tag{4}$$

where $\lambda_y = K_y / K_x$. $\lambda_{mm} \neq \lambda_{mr} \neq \lambda_{rm} \neq \lambda_{rr}$ in the coupled system while in the uncoupled system all four definitions are identical. What is also clear is that only λ_{mm} remains unchanged from the uncoupled to coupled systems (by the introduction of eccentricities) hence only this parameter can truly be named the uncoupled torsional frequency ratio. Thus the equation (1) defined at the CM has some advantages over CR when using λ_{mm} and r_m to normalise the rotations as both parameters are unaffected by the introduction of eccentricities to the problem. The transformed equation (1) at the CM would use $\left(\omega_x^2 = K_x / m, \omega_y^2 = K_y / m, \omega_{mm}^2 = K_{mt} / m r_m^2\right)$ frequencies parameters. The angular displacement $\varphi = r_m \theta$. The classical Rayleigh orthogonal damping matrix $[\mathrm{C}] = a_0[\mathrm{M}] + a_1[\mathrm{K}]$ where the damping constants a_0 & a_1 can be calculated from the resulting eigen-problem by assuming constant ratio of critical damping for the first and third mode.

APPLICATIONS OF GERSGORIN'S EIGENVALUE THEOREM

By a classical eigenvalue analysis, the system dynamic matrix defined at the CM as follows and thus by applying Gershgorin's Theorem the smallest eigenvalue μ must be as follows in (5). For dynamic structural stability the system must be positive definite, hence a conservative, but safe bound would be that all three conditions from (5) are greater that zero.

$$\mu \geq \min\left(1 - |\varepsilon_{my}|, \lambda_y^2 - \lambda_y^2 |\varepsilon_{mx}|, \lambda_{mm}^2 - |\varepsilon_{my}| - \lambda_y^2 |\varepsilon_{mx}|\right) \text{ hence } |\varepsilon_{my}| < 1, \ |\varepsilon_{mx}| < 1, \ \lambda_{mm}^2 > |\varepsilon_{mx}| \lambda_y^2 + |\varepsilon_{my}| \tag{5}$$

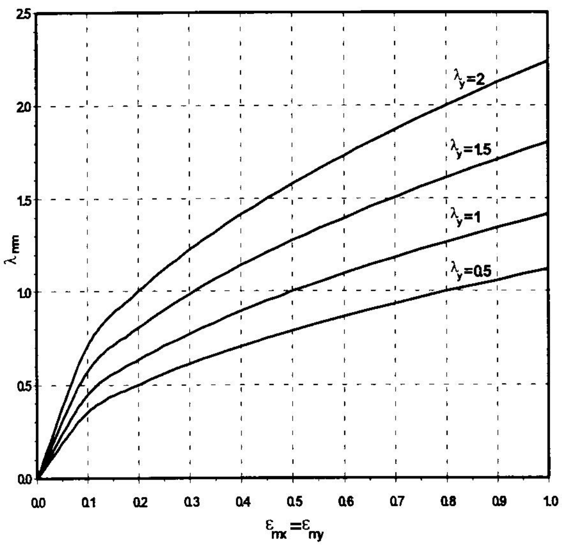

Figure 1

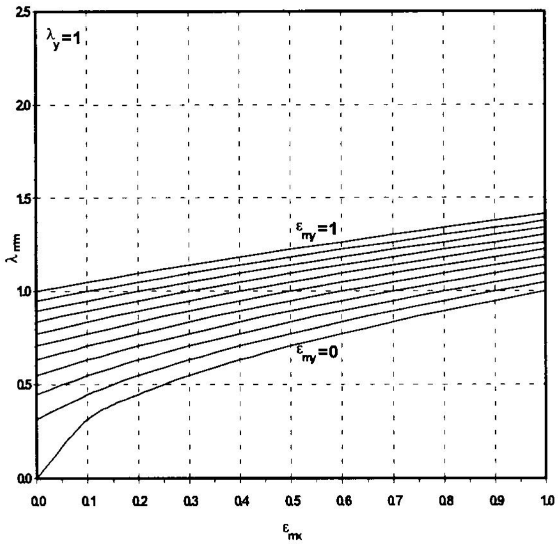

Figure 2

Thus the bounds on eccentricity ratios are that their absolute values are less than one. The third bound is interesting because it is a constraint on the relationship of the four system parameters. The eccentricities and the frequency ratios cannot take on any arbitrary values. If the eccentricities are equal then the figure 1 represents the lower bound for the torsional frequency parameter. If the lateral frequency ratio is set to unity then the figure 2 represents the lower bound for the torsional frequency ratio with respect to eccentricities. If an antisymmetric paraboloid function for load/deflection of the floor in the (x, y, φ) directions is conjectured then by application of the first three terms of a Taylor series expansion it is possible to state the following enhanced equation of motion (6) as (Alexander *et al* 1999) suggests. This model has maxima of stiffness forces occurring at approximately $x_m^2 + y_m^2 + \varphi^2 = \beta^2$. The nonlinearity is valid for deflections $x_m^2 + y_m^2 + \varphi^2 \leq 4\beta^2$. Equation (1) is parameterised and extended to include parametric elastic nonlinearity thus :

$$\begin{bmatrix} \ddot{x}_m \\ \ddot{y}_m \\ \ddot{\varphi} \end{bmatrix} + [C] \begin{bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\varphi} \end{bmatrix} + \frac{1}{2\beta} \begin{bmatrix} \omega_x^2 & 0 & -\omega_x^2 \varepsilon_{my} \\ 0 & \omega_y^2 & \omega_y^2 \varepsilon_{mx} \\ -\omega_x^2 \varepsilon_{my} & \omega_y^2 \varepsilon_{mx} & \omega_{mm}^2 \end{bmatrix} \begin{bmatrix} 2\beta - |x_m| + 2(\sqrt{2}-1)|\varphi| & 0 & 0 \\ 0 & 2\beta - |y_m| + 2(\sqrt{2}-1)|x_m| & 0 \\ 0 & 0 & 2\beta - |\varphi| + 2(\sqrt{2}-1)|y_m| \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ \varphi \end{bmatrix} = - \begin{bmatrix} \ddot{x}_g \\ \ddot{y}_g \\ 0 \end{bmatrix} \quad (6)$$

This elastic nonlinearity, while not modelling individual element elastic-plastic non-linear stiffnesses, does allow for a parametric analysis of a stiffness softening system. This parametric analysis is performed without reference to a particular structural system hence provides a mechanism for a more generalised investigation. The nonlinearity parameter varies with the period of the building hence.

$$\beta = \Delta_1 (10 T_x) \quad (7)$$

where Δ_1 is notionally the horizontal deflection of a single storey building at which the peak stiffness force is produced. In this study $\Delta_1 = 0.01$

GROUND MOTION PHASE-DIFFERENCE SPECTRUM

The role of phase content in accelerograms has been considered by various researchers (Kubo, T. 1984) (Katukura, H. *et al* 1989) (Yamanouchi H, *et al* 1990) concluding that phase content seems to be uniformly distributed between (0-2π) across the structural frequency range. Also that the overall envelope shape of the time domain accelerogram is closely related to phase content. Hence in the construction of synthetic accelerograms care must be taken in postulating the phase content. If, for any seismic event, the accelerograms are observed it becomes clear that peaks in the x and y directions are not synchronised. However there will be portions of the accelerograms where they are more synchronised than others This degree of synchronisation is an estimation of the phase difference content of a pair of x and y accelerograms. A more formal expression of this is to derive the Fourier frequency domain representation of the accelerogram pair. From this the phase content of the x and y history can be derived and thus the *phase difference spectrum* can be stated. A standard *radix-2* FFT algorithm is used. If zero end padding is performed, a spurious frequency content will be introduced. Thus the time series are truncated to 2^{11} data point. It can be shown that all the interesting strong motion reside within this limit. The cross-covariance function (un-normalised cross-correlation function is given by:

$$R_{xy}(\tau)=\int_{-\infty}^{\infty}\ddot{x}_g(t+\tau)\ddot{y}_g(t)dt\,,\quad S_{xy}(\omega)=\int_{-\infty}^{\infty}R_{xy}(\tau)e^{i\omega t}dt=\overline{H_x(\omega)}H_y(\omega) \tag{8}$$

$$\overline{H_x(\omega)}=\overline{\int_{-\infty}^{\infty}\ddot{x}_g(t)e^{i\omega t}dt}=a_x(\omega)-ib_x(\omega)=A_x(\omega)e^{-i\phi_x(\omega)},\quad H_y(\omega)=\int_{-\infty}^{\infty}\ddot{y}_g(t)e^{i\omega t}dt=a_y(\omega)+ib_y(\omega)=A_y(\omega)e^{i\phi_y(\omega)},$$

$$\overline{H_x(\omega)}H_y(\omega)=A_x(\omega)A_y(\omega)e^{i(\phi_y(\omega)-\phi_x(\omega))},\quad \phi_D(\omega)=\mathrm{Arctan}\left(\frac{a_x(\omega)b_y(\omega)-a_y(\omega)b_x(\omega)}{a_x(\omega)a_y(\omega)+b_x(\omega)b_y(\omega)}\right)=\phi_y(\omega)-\phi_x(\omega) \tag{9}$$

The Fourier transform of $R_{xy}(\tau)$ is known as $S_{xy}(\omega)$ the cross-spectrum (cross-spectral density function). This function has the property known as the correlation theorem. $\overline{H_x(\omega)}$ is the complex conjugate of $H_x(\omega)$. The cross-spectrum, $S_{xy}(\omega)$, is a complex function which can be evaluated from the Fourier integrals $H_x(\omega)$ & $H_y(\omega)$. The useful property of $S_{xy}(\omega)$ is that its phase spectrum (here known as the phase difference spectrum $\phi_D(\omega)$) is the difference of the phase spectrums of the x and y ground accelerations. Using the phase spectrum of the cross-spectrum $\phi_D(\omega)$ it is possible to state an interpolating formula for a proposed new phase content of the y accelerogram.

$$\phi_y^*(\omega)=\phi_x(\omega)+\psi\phi_D(\omega)\quad:\quad 0\le\psi\le 1 \qquad\qquad H_y^*(\omega)=\sqrt{a_y(\omega)^2+b_y(\omega)^2}\,e^{i\phi_y^*(\omega)} \tag{10}$$

Thus when phase difference parameter ψ = 1.0 the $\phi_y^*(\omega)$ is identical to $\phi_y(\omega)$ while if ψ = 0.0 then $\phi_y^*(\omega)$ is identical to $\phi_x(\omega)$. The y history is now reconstructed in equation (10) based on its original amplitude and its new phase $\phi_y^*(\omega)$ This reconstruction requires an inverse fast Fourier transform IFFT algorithm. In order to maintain a real function in the time domain the newly constructed $H_y^*(\omega)$ Fourier spectrum must have an even real part and an odd imaginary part. Note that since both $\ddot{x}_g(t)$ & $\ddot{y}_g(t)$ are real then by definition $a_x(\omega)$ & $a_y(\omega)$ are even and $b_x(\omega)$ & $b_y(\omega)$ are odd. Thus by inspection $\phi_D(\omega)$ is odd and hence $\phi_y^*(\omega)$ is also odd. So the newly constructed $H_y^*(\omega)$ spectrum will result in a real time domain function when it is inverse transformed. As the parameter ψ is varied the phase difference between the x and y accelerogram is modified.

TIMEHISTORY RESPONSE SPECTRUM STUDY

Corrected accelerograms were chosen from the US National Geophysical Data Center database of records. The selection criteria were (a) 5 horizontal accelerogram pairs from 4 soil class defined by Uniform Building Code. (b) magnitude 5-6 ML (c) 'near field' records with epicentral distance less than 30km. (d) mostly from ground floor buildings or free field records. (e) almost entirely from the USA. The selection of x and y components is problematic. Thus each accelerogram pair is used twice; once with the first accelerogram as the x component and the second accelerogram as the y. Then the accelerogram pair are swapped; the first accelerogram becoming the y component and second being x. Thus these 40 accelerogram pairs are normalised with respect to the group mean of the peak accelerations. This normalisation is also problematic as the amplitude ratio of x component to y component needs to be maintained. Hence the x component is normalised to the mean peak acceleration value and the y component is normalised to maintain the original x:y amplitude ratio. Given that for a particular accelerogram pair both orthogonal ground components are granted the opportunity to be the x component any bias is avoided. The structural parameters are varied thus: $0.8 \le \lambda_{mm} \le 1.2,\ 0 \le \varepsilon_{mx} \le 0.4,\ 0 \le \varepsilon_{my} \le 0.4,\ \lambda_y = 1.0$ Response time histories are used to construct the spectra of total acceleration responses of the structure(s).

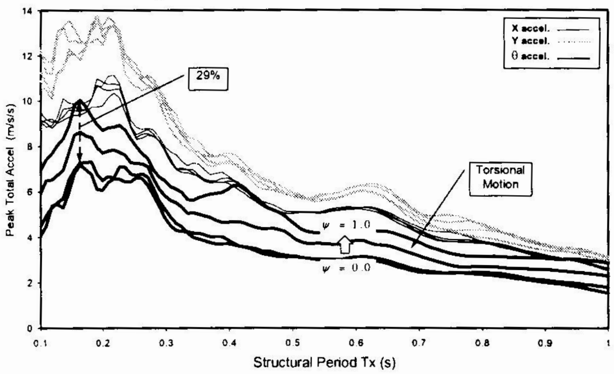

Figure 3: 95% spectral envelope for Soil Class 1

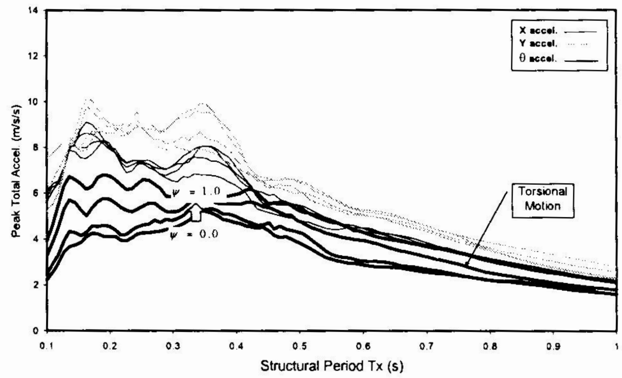

Figure 4: 95% spectral envelope for Soil Class 2

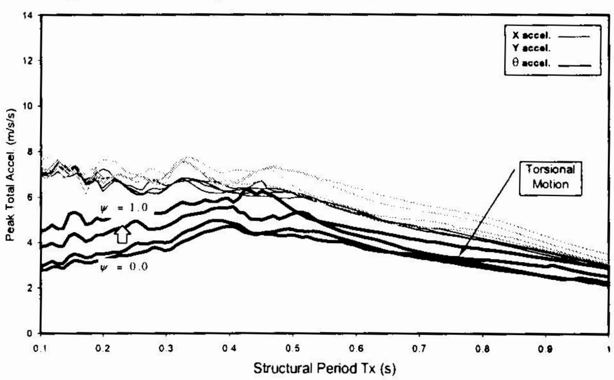

Figure 5: 95% spectral envelope for Soil Class 3

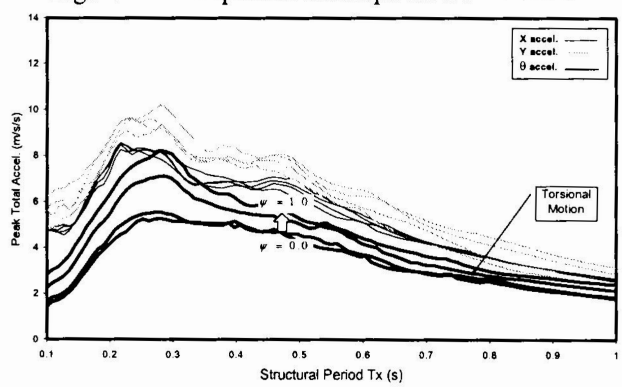

Figure 6: 95% spectral envelope for Soil Class 4

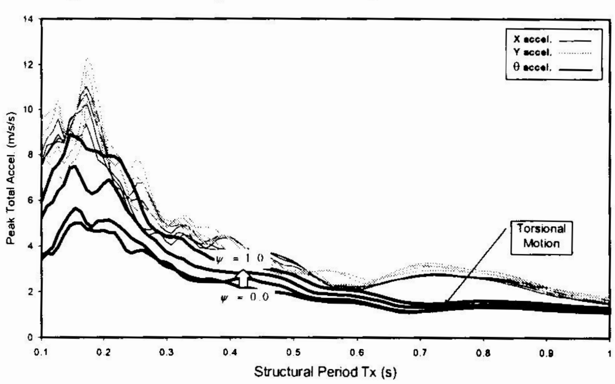

Figure 7: 95% Spectral envelope, Soil Class 1,
Whittier Narrows Earthquake, 1987

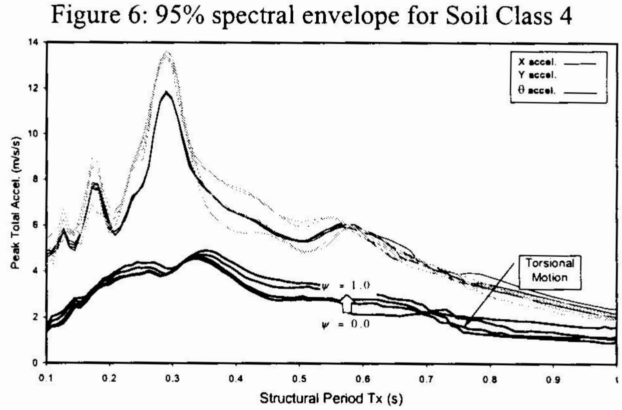

Figure 8: 95% Spectral envelope, Soil Class 1,
Alaskan subduction Earthquake, 1964

The mean μ and standard deviation σ of these spectra are calculated so that the $\mu + 1.645\sigma$ (95% spectral envelope) can be generated for the four soil classes. Figures 3 to 6 represent these 95% spectral envelope of responses with variation phase-difference parameter ψ for UBC soil classes 1 to 4.

DISCUSSION AND CONCLUSIONS

Varying the phase-difference ψ of the accelerograms while also varying the structural parameters (ie. eccentricties, structural frequencies etc.) has an effect on the x & y response accelerations, when looking at the 95% spectral envelope of responses. These 95% spectra represent a statistical envelope of responses for a range of structural configurations with variation in phase-difference. The general conclusion is that the removal of phase-difference of the two orthogonal horizontal ground components produces an approximately 10% increase in x & y response accelerations. This result is regardless of soil class. The effects on the torsional acceleration is more pronounced. A reduction in phase-difference between orthogonal ground components significantly reduces the torsional acceleration. The peak value of the torsional response accelerations from figure 3 is monitored with change in phase-difference. Thus a percentage variation in the peak torsional response can be stated. The soil class study indicates that the peak variation of the torsional acceleration response (with phase-difference variation) is about 29%,22%,28% & 34% for soil classes 1,2,3 & 4. If the "amount" of phase-difference between two orthogonal horizontal components can be assessed by considering the peak variation in torsional response then the "amount" of phase-difference seems not to be function of soil class. However, without a larger sample of accelerogram data, it is difficult to draw any firm conclusions. Also potential bias in the accelerograms due to the correction procedure has not been evaluated i.e. the effects of sampling, attenuation relationships etc. Figure 7 & 8 indicates that within the data for soil class 1 there is large variation. The percentage variation in peak torsional response is noted on figure 8 for these two earthquakes. The effects of varying the phase-difference on the Alaskan subduction Earthquake are small, about 4% variation in peak torsional acceleration. This indicates that the "amount" of phase difference for this accelerogram pair is small. Note also that the torsional acceleration is smaller than in figure 3. The effects of varying the phase-difference on the Whittier Narrows Earthquake is large, about 45% variation in peak torsional acceleration in figure 8. This indicates that the "amount" of phase-difference for this accelerogram pair is large as is the torsional response acceleration. There is change from a "twisting and swaying" building vibration mode when there is significant phase-difference in the ground motion to a "less twisting and more swaying" mode when there is no phase-difference in the ground motion. Hence care should be taken in the selection of accelerograms. Generally there is evidence of a phase-difference to torsional acceleration amplification relationship.

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